Problem 1:

It is irreducible since there is positive probability which transit from any state i to j.

It is a periodic since the period 

Also, we can see there exist a positive probability which can commute from state 1 to itself. Therefore the Greatest Common Divisor is 1. Periodicity is a class property, so this irreducible process is a periodic.

Since this Markov Chain is irreducible and aperiodic , the stationary probability will be the dominate eigenvector of the transition matrix, which can be obtain from the program:

>> nv

nv =

0.2500

0.2500

0.2500

0.2500

I use 10000 time steps to show the convergence.



Figure1: 4 sample paths of the Markov Chain with four states

y =

0.2548 0.2535 0.2536 0.2510

0.2501 0.2485 0.2483 0.2501

0.2463 0.2486 0.2499 0.2467

0.2486 0.2492 0.2480 0.2520

When state=1,

Relavtive\_Fre= 0.2548 0.2535 0.2536 0.2510

When state=2,

Relavtive\_Fre= 0.2501 0.2485 0.2483 0.2501

When state=3,

Relavtive\_Fre= 0.2463 0.2486 0.2499 0.2467

When state=4,

Relavtive\_Fre= 0.2486 0.2492 0.2480 0.2520

Comment:

From Figure 1, we can see the averages along the sample path for the 4 states are all converge to 0.25.

From the output, the averages along that sample path converges to the stationary probability for each state, which can be demonstrated that this result converges to 0.25 which we get for each state in Part(a).

>> avgleft

avgleft =

1.1261

>> right

right =

1.1250

Comment:

When simulate steps goes to infinity (take 10000 for example here), the left simulation result 1.1261, which converge to 1.125 we obtain by the formula.

Code:

clear all;

K=10000;

M=4; % 4 samples

N=4; % 4 states

% pi=[0.1 0.3 0.4 0.2;0.2 0.1 0.3 0.4;0.4 0.2 0.1 0.3;0.3 0.4 0.2 0.1];

pi= [0.1 0.3 0.4 0.2;0.2 0.4 0.0 0.4;0.0 0.3 0.5 0.2;0.5 0.3 0.2 0.0];

rand('seed',0);

for j=1:M

i=randsample(1:4,1);

for k=2:K-1

x(k+1,j)= randsample(1:4,1,true,pi(i,:));

i=x(k+1,j);

end

j=j+1;

end

for m=1:M

freq(:,m)=histc(x(:,m),1:N);

end

y=freq./10000;

plot(y);

% produce eigenvalues (D) and eigenvectors (V) of matrix PI

[V,D]=eig(pi');

ind=find(abs(diag(D)-1)<1e-6);

for k=1:length(ind)

% nv is the rescaled dominant eigenvector

nv(:,k)=V(:,ind(k))/sum(V(:,ind(k)));

end

f=[2.0 1.0 2.5 -1.0];

right = dot(f,nv);

x(1)=ceil(4\*rand);

for i=1:10000

[m,n]= sort(pi(x(i),:));

sign=0;

j=1;

u=rand;

while sign==0;

if u<sum(m(1:j));

x(i+1)=n(j);

sign=1;

else

j=j+1;

end

end

end

for i=1:10000

left(i)=sum(f(x(i)));

end

avgleft=sum(left)/10000;

Problem 2:

It is irreducible since there is positive probability which transit from any state i to j.

There exists a positive probability which can commute from state 1 to itself. Therefore the Greatest Common Divisor is 1. Periodicity is a class property, so this irreducible process is a periodic.

Since this Markov Chain is irreducible and aperiodic , the stationary probability will be the dominate eigenvector of the transition matrix, which can be obtain from the program:

>> nv

nv =

0.1975

0.3333

0.2469

0.2222

I use 10000 time steps to show the convergence.



Figure2: 4 sample paths of the Markov Chain with four states

y =

0.1937 0.1927 0.1938 0.1995

0.3278 0.3361 0.3345 0.3242

0.2498 0.2496 0.2462 0.2592

0.2285 0.2214 0.2253 0.2169

When state=1,

Relavtive\_Fre= 0.1937 0.1927 0.1938 0.1995

When state=2,

Relavtive\_Fre= 0.3278 0.3361 0.3345 0.3242

When state=3,

Relavtive\_Fre= 0.2498 0.2496 0.2462 0.2592

When state=4,

Relavtive\_Fre= 0.2285 0.2214 0.2253 0.2169

Comment:

From Figure 2, we can see the averages along the sample paths for the 4 states are all converge to the dominant eigenvector nv.

From the output, the averages along that sample path converges to the stationary probability for each state, which can be demonstrated that this result converges to the probability 0.1975, 0.3333, 0.2469, 0.2222, which we get for each state in Part(a).

>> avgleft

avgleft =

1.1253

>> right

right =

1.1235

Comment:

When simulate steps goes to infinity (take 10000 for example here), the left simulation result 1.1253, which converge to 1.1235 we obtain by the formula.

Problem 3: